Developing teachers' attention to students' algebraic reasoning

Ana C. Stephens
Laura Grandau
Pamela Asquith
Eric J. Knuth
Martha W. Alibali

University of Wisconsin-Madison

American Educational Research Association
San Diego, CA
April 2004

Developing teachers' attention to students' algebraic reasoning

Research on the teaching and learning of algebra has recently been identified as a priority by members of mathematics education research community (e.g., Ball, 2003; Carpenter & Levi, 2000; Kaput, 1998; Olive, Izsak, & Blanton, 2002). Rather than viewing algebra as an isolated course of study to be completed in the eighth or ninth grade, these researchers advocate the reconceptualization of algebra as a strand that unifies the entire K-12 curriculum.

Most work on this "reconceptualization" has occurred in the early elementary grades. Researchers have identified core algebraic ideas such as equality (Carpenter, Franke, & Levi, 2003), operation sense (Schifter, 1999), and generalization (Blanton & Kaput, 2003) that students can begin to engage with as early as the first grade. Although longitudinal studies have not been conducted to examine the impact of such engagement on students' later success with traditional high school algebra courses, the assumption is that a focus on such core concepts will strengthen students' capacity for more advanced algebraic thinking in these later grades.

While the elementary grades offer a natural starting place for this work, we argue that unique challenges often exist in the middle grades in terms of reforming both the learning and teaching of algebra. Students in these grades have in many cases not experienced an "algebrafied" K-5 curriculum emphasizing notions of equality, operation sense, and generalization. There is some sense of urgency, then, to help students develop these important understandings in a shorter period of time while addressing the existing curricular demands of the middle grades.

Working with middle-school teachers on issues of algebra reform also presents a special set of challenges. While it is the case that elementary teachers must often teach from prescribed curricular materials, such materials are rarely described or even alluded to in the early algebra literature. Carpenter, Franke, and Levi's (2003) accounts of elementary students engaged in discussions around the meaning of the equal sign, relational thinking, and conjecturing and justifying, for example, provide images of teachers with great freedom to choose the content of their mathematics courses. No mention is made of the realities of prescribed curricula experienced by many teachers.

While teacher content knowledge is a concern across the grades, the increasing complexity of the mathematics in the curriculum places even more demands on middle-school teachers. And, as is the case at all grade levels, teacher content knowledge varies widely in the middle grades, with mathematics specialists (often seventh- and eighth-grade teachers), generalists (often sixth-grade teachers), and special education teachers all involved in the teaching of mathematics. Such variance in content knowledge can make professional development work both a challenge and an opportunity for teachers to learn from each other.

Framed by the unique features and challenges of algebra reform in the middle grades, this paper will describe a research project aimed at exploring the opportunities middle-school teachers have to engage their students in algebraic thinking when supported by a particular professional development experience. This professional development experience—provided by the University of Wisconsin-Madison's *Supporting the Transition from Arithmetic to Algebraic Reasoning (STAAR)* team—is one that, to borrow Blanton and Kaput's (2003) phrase, is focused on developing teachers' "algebra eyes and ears." Teachers with developed algebra eyes and ears are ones who are able to recognize the potential offered by tasks to engage students in algebraic thinking, are able to recognize algebraic thinking demonstrated by students, and are able to elicit such thinking through question-posing and task extension. After providing background information about our research questions and professional development participants, structure, and goals, we will provide an image of the professional development experience of our teachers, discuss progress towards our research questions and professional development goals, and highlight some of the challenges we experience in conducting this work.

Research questions

Our overall research purpose is to explore the opportunities that exist for engaging students in algebraic reasoning in the context of middle-school mathematics classrooms and the extent to which teachers recognize and capitalize on these opportunities. To that end, our specific questions are the following:

1. To what extent do teachers recognize the potential offered by tasks to engage students in algebraic thinking?

1. To what extent do teachers recognize algebraic thinking demonstrated by students?

1. To what extent do teachers modify tasks/lessons in order to take advantage of opportunities to engage students in algebraic thinking during instruction?

Investigating these questions has involved the collection of multiple forms of data, including video documentation of professional development activities and classroom observations with three participating teachers. In addition to providing a research focus, these questions have also framed our professional development work with teachers.

The purpose of this paper is to provide an image of the professional development experience of our teachers, discuss connections we have identified between this experience and two participating teachers' practices, and share some of the challenges we have faced in our pursuit of the development of middle-school teachers' algebra eyes and ears. We begin by describing in more detail our professional development participants, structure, and goals.

Professional development participants and structure

Professional development participants include 16 teachers from two socio-economically diverse middle schools in a small urban district. Six of these teachers are special education teachers and the remaining are regular classroom teachers. Levels of teaching experience range from 2 to 30 years. A learning coordinator from one school and two district-wide instructional resource teachers are also in attendance. The majority of these participants have spent the past two years taking part in our professional development, which each year has consisted of a three-day summer workshop and ten academic-year after-school sessions.

It should be noted that while we have engaged our teachers in professional development over the past two years, teachers' knowledge and practices have only been a focus of research over the past one year. We first started our professional development work two years ago because we wanted to give something back to the teachers of students whom we were studying. "Giving back" involved sharing data we had collected and analyzed regarding their students' understandings of core algebraic concepts and providing short, related tasks they could pose to their students to further explore their thinking. It was not until the beginning of the second year that we developed the previously-presented set of research questions and a plan to systematically examine relationships between our professional development and teachers' practices. Thus what we discuss here is not a story of change in teachers' knowledge or practices across time but rather a story of our attempts to increase teachers' attention to students' algebraic thinking and opportunities for engaging students in such thinking during our second year of professional development.

Professional development goals and activities

As stated earlier, our overall professional development goal is to contribute to the development of our middle-school teachers' algebra eyes and ears. We draw from Blanton and Kaput's (2003) "algebrafication" strategy which focuses on three types of teacher-based change: (a) algebrafying curricular materials, (b) recognizing and supporting students' algebraic thinking, and (c) creating teaching practices that promote algebraic thinking.

A variety of professional development activities have taken place in pursuit of these goals, including teacher engagement in problem-solving activities, the study of student work, and the examination of curricular materials for opportunities to engage students in algebraic thinking. We have, for example, asked teachers to solve particular tasks and then think about how they might be extended to engage students in even further algebraic thinking, to think about how their students might solve such tasks, and to pose tasks to their students to investigate their thinking. With one still-ongoing project, teachers have been asked to select a lesson from a "non-algebra" strand of their curriculum, *Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998), look for opportunities to algebrafy the lesson, teach the modified lesson to their students, and then discuss with colleagues the results of this algebrafication.

As described above, our work with teachers has often been very task- or lesson-specific. Our overall professional development goal, however, is not one that is tied to particular tasks or lessons. We hope to help teachers develop algebra eyes and ears so they will be better able to recognize and capitalize on opportunities to engage students in algebraic thinking as they occur in their day-to-day lesson planning and classroom interactions. Thus the individual tasks we share with teachers are not in themselves of particular importance, but we hope that providing a common opportunity to examine these individual tasks and students' algebraic thinking in response will contribute to the development of teachers' algebra eyes and ears more broadly so that they may recognize and capitalize on more "naturally" occurring opportunities to engage students in algebraic thinking.

In what follows, we describe a sequence of professional development activities in which we took such a task-specific approach to provide teachers with a focused space to think about opportunities for engaging students in algebraic thinking, in this case around issues of variable and representational fluency. We provide a detailed description of this sequence of activities to illustrate our professional development efforts aimed towards developing our middle-school teachers' algebra eyes and ears and the successes and challenges we continue to experience. Again, we stress that this task-specific approach was taken to provide teachers with an example of what can happen when one thinks deeply about opportunities to engage students in algebraic thinking. The particular task that will be discussed served as a vehicle for helping teachers "practice" looking for students' algebraic thinking and opportunities to push this thinking. We first provide an overview of the sequence and justification for its individual components.

A professional development sequence

The sequence we designed around a common mathematical focus included multiple practices supported by the teacher professional development literature: sharing research data on students' mathematical thinking, engaging teachers in mathematical problem solving, asking teachers to implement a particular mathematical task in their own classrooms, and analyzing students' written work.

First, teachers were presented research data on students' algebraic thinking generated by the *STAAR* project. We implemented this strategy in response to findings of Cognitively Guided Instruction (CGI) researchers (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) that the sharing of research data on students' thinking in the domain of whole-number arithmetic resulted in changes in teacher beliefs and practices. Teachers with such knowledge tended to spend more instructional time on problem solving, encourage students to use a variety of problem-solving strategies, and listen to their students' problem-solving processes significantly more than did control teachers. While frameworks describing student thinking in the domain of algebra are not nearly as constrained or robust as those presented in CGI's professional development, we nonetheless believed that the sharing of *STAAR* research findings in this domain had the potential to motivate teachers' own "practical inquiry" (Franke, Fennema, & Carpenter, 1997) focused on how their students think about and solve specific mathematical problems.

Second, after *STAAR* data were shared, teachers were presented with a related task and were asked to engage with the mathematics of the task. This is a point at which our "algebrafication" strategy differs from Kaput and Blanton's (2003). Rather than having an algebraic idea (e.g., generalization) lead to the design of a task, we used our knowledge of student thinking as a starting point and provided a task that had the potential to confront an observed misconception. The assumptions underlying our focus on *teacher* problem-solving include that teachers benefit from experiences as students that are based on the same principles as the ones they are expected to implement with students (Loucks-Horsley, Hewson, Love, & Stiles, 1998). That is, both students and adults learn by constructing their own meanings using previous knowledge. If teachers are to interact with their students in ways that encourage them to construct their own problem-solving strategies and mathematical representations of problem situations, they must first fully understand this process for themselves.

Third, teachers were asked to implement the same task with their students. The motivation for doing so comes from both the closely-related presentation of *STAAR* research data and the fact that teachers just engaged in problem-solving and discussion with colleagues on the same task. We asked teachers to think about what their students might do with the task, and this third phase of the sequence provided the opportunity for them to find out.

Finally, teachers were asked to bring students' written work on the task for analysis at the following meeting. Many see this professional development strategy as the most powerful way to help teachers improve their practice. As Loucks-Horsley et al. (1998) argue, "Student learning is the ultimate outcome for professional development, and the closer the professional development opportunity bring teachers to student learning the better" (p. 121). Students' written products can reflect their thinking and help their teachers gain insights into this thinking. We now describe in more detail the particulars of the four-part sequence we implemented—one motivated by a focus on students' understandings of variable.

Presentation of research data. *STAAR* data were presented on students' success solving simple algebraic equations and, more specifically, on their interpretations of variables. For example, when sixth- through eighth-grade students (n = 371) at one of the schools represented by our professional development teachers were asked "Is $h + m + n = h + p + n$ always, sometimes, or never true?" less than half in each grade correctly responded "sometimes." A significant number responded "never." A few elaborated by writing "p is a different number than m" or "m and p can't be the same number."

To examine the same issue, 16 sixth-graders (a subset of the above group) were shown the number sentences $a = a$, $c = r$, and $c = r + t$ and were asked whether they were true or false and why. Most students (13 out of 16) correctly stated that $a = a$ was true because, as one student explained, "The variable a has to be the same in the same problem." Only 3 out of 16 students, however, correctly stated that $c = r$ could be true or could be false, depending on the values of these variables. The majority of students held the belief that $c = r$ must be false, because, as one representative student stated, “When a letter represents a number usually each letter represents a different number, not the same ones.” Ten out of the 16 students correctly stated that $c = r + t$ could be true or could be false. The interesting point to note here is that students can respond correctly to this prompt while holding the misconception that different letters must represent different numbers. One student even stated that if c were equal to 20, then r and t could stand for 5 and 15 but could not stand for 10 and 10.

The data described above, accompanied by videos of a few of the interviews, were presented to teachers over the course of two professional development meetings. Several teachers had questions about the structure of the interview (e.g., Did we tell students after each question whether or not they were right? Why didn’t we offer “can’t tell” as an explicit option?) Our findings did not appear to surprise the teachers a great deal, given the sixth-grade students’ limited exposure to symbolic representations of variables. That the values of c and r in $c = r$ can in fact be equivalent is a mathematical convention, not a notion that is intuitively obvious. Rather than simply being asked to memorize it, however, students should engage with problem situations that support the adoption of this convention. The mathematical task we presented to our teachers next introduced one such situation.

Teacher engagement with mathematics. Following the sharing of the *STAAR* interview data, the following problem, adapted from Carpenter, Franke, and Levi (2003) was posed: *Ricardo has 8 pet mice. He keeps them in two cages that are connected so that the mice can go back and forth between the cages. One of the cages is blue and the other is green. Show all the ways that 8 mice can be in two cages.*[1]

---

[1] The problem as presented in Carpenter, Franke, and Levi (2003) is as follows: “Ricardo has 7 pet mice. He keeps them in two cages that are connected so that the mice can go back and forth between the cages. One of the cages is

The teachers solved this problem on their own and then shared their representations of the situation. Most teachers approached the problem by creating a systematic table illustrating the nine possibilities (e.g, 0 mice in the green cage and 8 mice in the blue cage, 1 mouse in the green cage and 7 mice in the blue cage, and so on). One teacher thought about how many combinations would exist if he “named” the mice. He found that if the mice were treated as individuals, there would be one way to have 0 mice in the green cage and 8 mice in the blue cage ($_8C_0$), eight ways to have 1 mouse in the green cage and 7 mice in the blue cage ($_8C_1$), and so on, for a total of 256 combinations. Not all teachers were comfortable with the mathematics of his approach, even after his explanation and some discussion. One teacher joked, “The smart kid said it, so it must be right.” This highlighted for us the challenge of having varying degrees of content knowledge in the group. A solution accessible to an eighth-grade teacher who is a subject-matter expert was not accessible to many of the others in the group.

Few if any of the teachers represented the situation with an algebraic equation. While this was not surprising given the task did not ask for such a representation, we had hoped to discuss the task in terms of student thinking about variables. After the sharing of solutions, we asked teachers to think about how this problem might relate to the data on students’ understandings of variable we had shared earlier. With some prompting, teachers noted that the sum of the numbers of mice in the two cages was always equal to 8 and that the situation could be represented by the equation $x + y = 8$. We then asked how this task might be used to confront the demonstrated misconception and one teacher pointed out that in the case of 4 mice in each cage, the values of x and y would be the same. The teachers were then asked to pose the *Mice Problem* to their students and bring the resulting written work to our next meeting to share with their colleagues.

---

big and the other is small. Show all the ways that 7 mice can be in two cages” (p. 65). The change from 7 mice to 8 mice was made in order to allow for the possibility of having the same number of mice in each cage.

Task implementation in classrooms. We observed and videotaped the classrooms of three of our seventh-grade teachers as they posed the *Mice Problem* to their students in the weeks following and will describe two of our observations here.

Sarah asked her students to work on the Mice Problem in groups of two and to write their solutions on a whiteboard to share with the rest of the class. Sarah, as we have observed her do on other occasions, asked her students to find more than one suitable representation of the problem situation. As she walked around the room, she pushed pairs of students to think about how they could represent the situation using variables. After students had spent some time working on the task, Sarah asked students to take turns sharing their solutions. The representations generated included pictures, tables, and one graph (see figure 1). Although algebraic equations surfaced in students' written work (see figure 2), this representation was not shared with the whole group at this time. After students' solution-sharing time was complete, Sarah mentioned that she noticed some of them had written the equation $x + y = 8$. She then engaged students in a discussion closely connected to the events of our last professional development meeting. She asked students, "Can $x = y$ be true?" Some students initially said "no," some students said "yes," and some students said "sometimes." One student explained that it was true when there were 4 mice in each cage. Sarah then posed the interview tasks ($a = a$, $c = r$, and $c = r + t$) we had shared at our last meeting and asked students whether these were true or false. Most students appeared comfortable with the fact that c and r, for example, could stand for the same numbers or for different numbers.

What we see in Sarah's classroom is a teacher who has already developed one aspect of mathematics reform teaching. Sarah feels very comfortable asking her students to share problem-solving strategies and probing their understandings through questioning. We see Sarah eliciting algebraic thinking by asking her students to think of multiple representations of the given problem situation, including variable representations. Simply posing the *Mice Problem* to students and asking them to share their strategies would not have produced the depth of thinking about variable and multiple representations that we observed in Sarah's classroom. Her recognition of the algebraic opportunities offered by the task was instrumental in shaping her students' experiences with it. In a post-observation interview, Sarah said she wanted her students to "see if they could see the pattern and express it algebraically," meaning with variables, as well as with other representations. We also see a clear connection to our professional development in Sarah's posing of the interview tasks $a = a$, $c = r$, and $c = r + t$. What we do not yet see is a larger discussion addressing the connections *among* students' solution strategies. In the case of the *Mice Problem*, this might have involved examining the table, the graph, and the equation side-by-side and discussing the connections among them, the advantages one might have over another, the information that is most apparent in each representation, and so on. We now present a description of Karen's classroom, where we did in fact see some connections among representations being made.

Karen's implementation of the *Mice Problem* began in much the same way as Sarah's, except that her students worked individually. While they worked, she circulated around the room helping those who were struggling. The student work collected in Karen's class suggests that almost all students were able to generate pictorial or tabular representations but that none spontaneously generated a graph or an algebraic equation. The class discussion that followed in Karen's class was unlike the one that took place in Sarah's classroom. Rather than having several students come to the front of the room to share their strategies, Karen took a more direct approach. She started by drawing a table on the board and asking her students to provide the data points within the table. She arranged their responses in a systematic way, ranging from 0 mice in the blue cage and 8 mice in the green cage at the top of the table down to 8 mice in the blue cage and 0 mice in the green cage at the bottom of the table. Karen then helped her students move from this representation to a symbolic one by asking, “What do you notice in this table? Do you see any patterns?” One student noted that the numbers down one column of the table decreased while those down the other column increased. Another student noted that the numbers in each row of the table summed to 8. When Karen asked the students if they could think of an equation that could be written to describe the situation, the first one proposed was $8 \div 2 = 4$. When asked to think of an equation that would apply to *all* possibilities, one student proposed $8 - g = b$. From this equation, the students were able to generate the equivalent $8 - b = g$, $b + g = 8$, and $g + b = 8$. The students agreed that these four equations were true for all of the data generated for the *Mice Problem*. Karen then asked her students what they thought about the statement $b = g$. They quickly agreed that it was sometimes true. As in Sarah’s class, one student pointed to the case of 4 mice in each cage. Karen stressed the point that in this case, two different letters—*b* and *g*—represented the same number. In the pre-observation interview, Karen mentioned that helping students understand this point was one of her major goals. In the post-observation interview, she stated that she was influenced by the student interview videos shown in our professional development and saw a relationship between the *Mice Problem* and the true-false items.

In the case of Karen we see explicit connections being made between different mathematical representations. Karen helped students generate an equation using variables by starting with the tabular representation—one that most of them already understood. The discussion of the possible values of *b* and *g* that followed was strengthened by having the two representations side by side. When a student stated that *b* was equal to *g* when you have 4 mice in each cage, Karen drew a circle around the row of the table in which this occurred. Thus we see Karen—at least in this example—moving beyond multiple representations to multiple representations with discussion of connections among them.

In both Sarah and Karen's classrooms we see connections being made to a discussion previously held in our professional development course about the possibility offered by the *Mice Problem* to confront a common misconception about variables. On some level, then, we know that these teachers are taking aspects of their professional development back to the classroom. Whether their work on this specific task will contribute to our broader goal of helping these teachers take advantage of day-to-day opportunities to engage students in algebraic thinking remains an open question. We conjecture that experiences such as the one presented are beneficial in providing teachers a focused space in which to think about this broader goal.

Analysis of student work. Teachers came to the next professional development meeting—approximately 6 weeks later—with student work on the *Mice Problem* and engaged in small group discussions around this work. As did Sarah and Karen, teachers across the grades found that most students approached the *Mice Problem* by drawing pictures of the two cages and illustrating the different numbers of mice that could be in each or by creating tables listing the possible combinations of mice in the blue and green cages. They found that many students were able to determine that there were nine possible ways to distribute 8 mice in two cages but that several students did not take the possibility of having 0 mice in one cage and 8 in the other into account.

One group of teachers tallied and presented the number of students who created each representation at each grade level. While this type of analysis had been modeled with previously-presented tasks, this was the first time we observed teachers spontaneously engaging with it.

Teachers said they liked the task because it provided opportunities for students to work with multiple representations. They also appreciated its accessibility. In this regard, we discovered that asking teachers to pose the same task across different grades has its benefits and drawbacks. There is often much variation in student ability within just one classroom, but across classrooms with students from three different grade levels, task accessibility requires even greater consideration. In the case of the *Mice Problem*, teachers felt that students of all abilities had immediate access to the task and could make at least some progress towards a solution.

After teachers discussed their students' written work, we showed video clips from the classrooms of Sarah, Karen, and one other participating teacher and asked teachers to identify key instances that stood out for them. Karen's video generated the most discussion. Teachers liked her orchestration of the $b = g$ discussion and the way in which she built to that point. One teacher was interested in the way Karen encouraged a systematic rather than random creation of the tabular representation. Another teacher noted her surprise at how many equations the students were able to generate.

Outside of the discussion of Karen's video, teachers did not address the potential this task offered to discuss with students the values the variables in a symbolic representation could take on, as was stressed at the previous month's meeting. Teachers viewed this task as one about multiple representations and "seeing what kids would do" rather than about providing a forum to discuss a particular mathematical issue. It was rather clear that with the exception of Sarah and Karen, our discussion about students' conceptions of variable did not appear to have made its way into the teachers' classrooms. We believe this occurred because many teachers treated the task as a data-collection opportunity (with the intent of gathering student work to discuss with colleagues *later*) rather than as an opportunity to engage students in discussion. This is likely due to the fact that the *Mice Problem* did not necessarily fit into the teachers' ongoing plans and may have been viewed as an interruption—albeit a potentially interesting one—in the curriculum. We in fact did not insist that teachers implement the *Mice Problem* with their students in a particular way but rather let them decide how much time they wished to devote to the task. For many teachers, this meant implementing the task as a short "warm up" with little or no discussion with students. We need to think about how to respond to this disconnect between our tasks and the teachers' ongoing instructional plans in ways that will encourage rich discussions among teachers and students. While, as we have argued, providing teachers a specific task on which to focus provides a space for them to consider questions of algebraic opportunity and share findings with colleagues who have implemented the identical task, this activity necessarily interrupts teachers' ongoing plans. How ones deals with this tension in a productive way is a question with which we are currently struggling.

Conclusion and next steps

As a reminder to the reader, we again list our overarching research questions:

1. To what extent do teachers recognize the potential offered by tasks to engage students in algebraic thinking?

2. To what extent do teachers recognize algebraic thinking demonstrated by students?

3. To what extent do teachers modify tasks/lessons in order to take advantage of opportunities to engage students in algebraic thinking during instruction?

In response to these research questions, the discussion of our professional development sequence points to a few instances in which teachers' algebra eyes and ears are being exercised—and perhaps even developed.

Our classroom observations indicate that, when provided the support of our professional development activities, some teachers did recognize the potential offered by tasks to engage students in algebraic thinking and took advantage of these opportunities during instruction. Both of the teachers described here listened to their students' strategies and encouraged multiple representations of the problem situation (pictures, tables, graphs, and symbolic equations). One of these teachers helped her students not only generate and make sense of multiple representations but also recognize connections among them. While these teaching practices cannot necessarily be attributed to our professional development, we did observe that both teachers made direct connections back to our discussion of variable in their discussions with students. The sharing of our *STAAR* data and discussion of the related mathematics behind the *Mice Problem* clearly had an impact on the way in which the task was discussed in these classrooms and contributed to Sarah and Karen's abilities to capitalize on the algebraic opportunities offered by the task.

The question of what long-term consequences the *Mice Problem* sequence and sequences around other tasks will have on teachers' practices remains largely unanswered. Will providing the opportunity for teachers to examine in depth the algebraic potential offered by an isolated task translate into their examining the algebraic potential offered by more "routine" tasks from their curriculum? Will looking for algebraic thinking in students' written work and verbal participation around a particular task help teachers look for such thinking and capitalize on such thinking in the course of their day-to-day work with students? We conjecture that it will, but this in fact largely remains to be seen.

We close by mentioning further data we are in the process of collecting and analyzing that we believe will contribute to our abilities to further address our research questions. First, we have conducted semi-structured interviews with ten participating teachers (including the three who have been the focus of our classroom observations) in which they were asked to describe what it means to think algebraically, to examine tasks and comment on their potential to engage students in algebraic thinking, and to examine student work on these same tasks and comment on the algebraic thinking observed. We believe analysis of this data set will help us address our first two research questions and provide insight into teachers' knowledge that can inform our professional development efforts.

Second, as mentioned in our description of professional development activities, teachers have been engaged in an ongoing project involving the "algebrafication" of a lesson from their curriculum. Our last two professional development meetings of the current academic year will provide teachers a forum to share their modified lessons and their observations of student thinking during the implementation of these lessons. While this is, again, a very task-specific exercise, teachers' modified lessons and presentations may provide some insight into our third research question as well as the first two research questions.

Finally, as the *STAAR* project concludes its third year, it is time to reflect on what we have learned about the professional development of middle-school mathematics teachers over the past two years and how we can scale up our efforts to contribute to the development of algebra eyes and ears for a larger number of teachers.

References

Ball, D. L. (Ed.). (2003). *Mathematical proficiency for all students: Toward a strategic research and development plan in mathematics education*. Santa Monica, CA: RAND Corporation.

Blanton, M. L., & Kaput, J. J. (2003). Developing elementary teachers' "algebra eyes and ears". *Teaching Children Mathematics, 10*(2), 70-77.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.-P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499-531.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in the elementary school*. Portsmouth, NH: Heinemann.

Carpenter, T. P., & Levi, L. (2000). *Developing conceptions of algebraic reasoning in the primary grades*: National Center for Improving Student Learning and Achievement in Mathematics and Science. University of Wisconsin-Madison.

Franke, M. L., Fennema, E., & Carpenter, T. P. (1997). Teachers creating change: Examining evolving beliefs and classroom practice. In E. Fennema & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 255-282). Mahwah, NJ: Lawrence Erlbaum.

Kaput, J. J. (1998). Transforming algebra from an engine of inequity to an engine of mathematical power by "algebrafying" the K-12 curriculum. In S. Fennel (Ed.), *The nature and role of algebra in the K-14 curriculum: Proceedings of a National Symposium* (pp. 25-26). Washington, DC: National Research Council, National Academy Press.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (1998). *Connected Mathematics*: Dale Seymour Publications.

Loucks-Horsley, S., Hewson, P. W., Love, N., & Stiles, K. E. (1998). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Sage.

Olive, J., Izsak, A., & Blanton, M. (2002). Investigating and enhancing the development of algebraic reasoning in the early grades (K-8): The Early Algebra Working Group. In D. S. Mewborn, P. Sztajn, D. Y. White, H. G. Wiegel, R. L. Bryant, & K. Nooney (Eds.), *Proceedings of the twenty-fourth annual meeting of the international group for the psychology of mathematics education* (Vol. 1, pp. 119-120). Columbus, OH: ERIC.

Schifter, D. (1999). Reasoning about operations: Early algebraic thinking in grades K-6. In L. V. Stiff (Ed.), *Developing mathematical reasoning in grades K-12* (pp. 62-81). Reston, VA: National Council of Teachers of Mathematics.

Figure 1

Three students' representations of the Mice Problem.

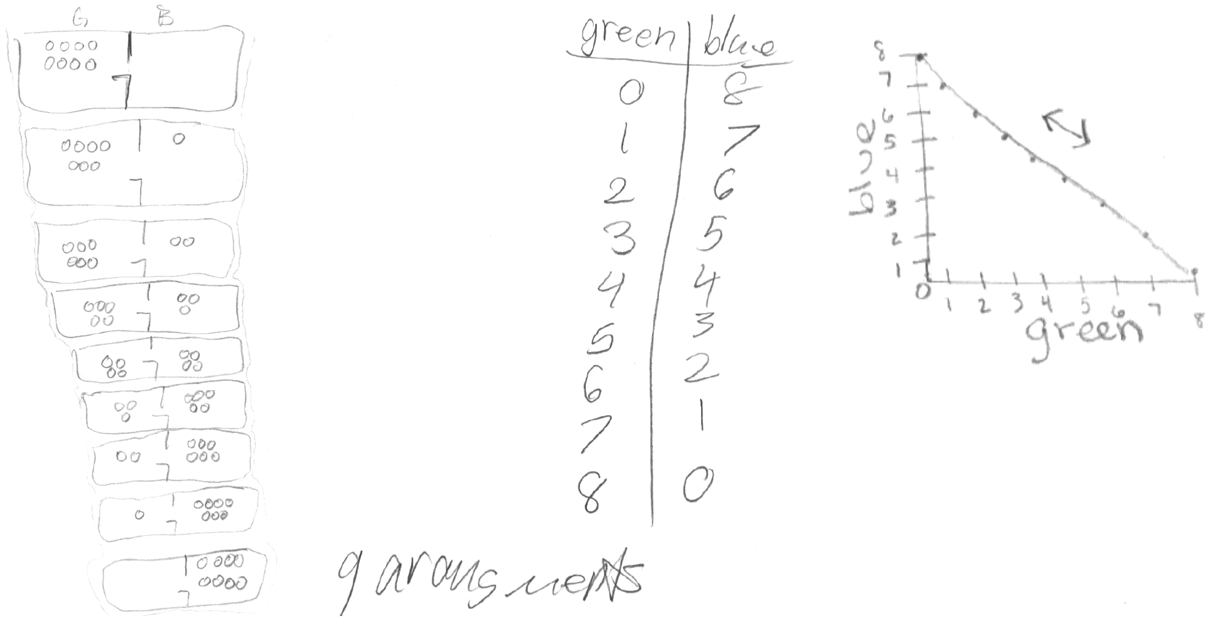

Figure 2

One student's symbolic representation of the Mice Problem.